PERIOD-IV

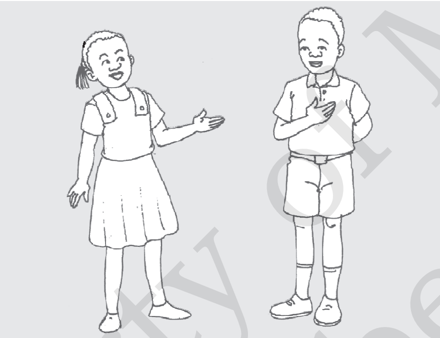
TOPIC 7

Relations and Functions

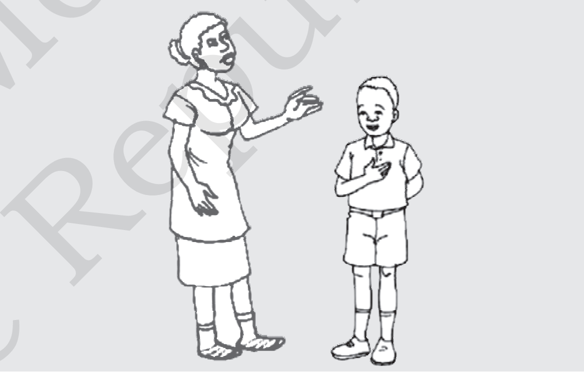
7.1 RELATIONS

In our daily life, we come across many patterns that characterize relations such as brother and sister, father and son, teacher and pupil, etc. A ‘relation is just a relationship between two sets of information’.

For example:



Ella “is the sister of” Felix.



Jerelyn “is the teacher of” Samuel.

Here, the pairing of the names of first person and second person is a relation. In these relations, the pairs of names of first person and second person are ‘*ordered*’, which means one comes first and the other comes second and this order of each pair cannot be changed.

Realizing the Relations in Mathematics

In mathematics also, we come across many relations such as the *numeral* 7 “is less than” the *numeral* 11, line l “is parallel to” line m, set A “is a subset of” set B.

Consider the following examples of relation:

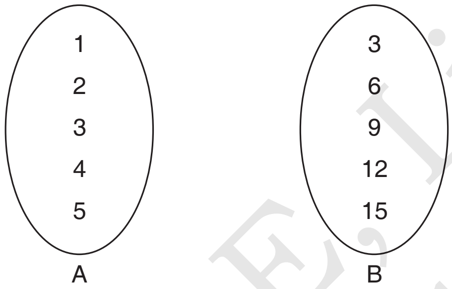
2 “is half of” 4. 3 “is the square root of” 9.

5 “is less than” 8.

$\frac{8}{10}$ "is an equivalent fraction of" $\frac{4}{5}$.

In all these examples, we notice that a relation involves pairs of objects in certain order.

Example 1. *Find the relation between the following sets by considering the numbers opposite to each other:*



Solution. By considering the numbers opposite to each other, we find that

1 is one third of 3, 2 is one third of 6,
3 is one third of 9, 4 is one third of 12,
5 is one third of 15.

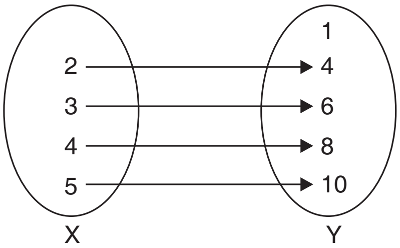
Thus in general, every element of set A is one third of its opposite element in set B.

The relation is therefore "is one third of".

Representing a Relation as a Mapping

A *relation* can be represented by matching diagram.

Consider the following matching diagram:



What do you observe?

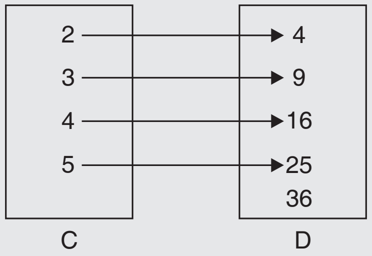
2 is half of 4, 3 is half of 6,
4 is half of 8, 5 is half of 10.

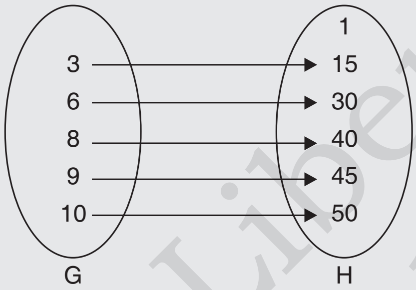
Thus in general, every element of set X is half of its corresponding element in set B.

The relation is therefore "is half of".

ACTIVITY 1

Consider the following relations:





What are the two sets shown?

What is the relation between the two sets C and D?

What about elements in set G and set H?

We observe that in the sets C and D:

2 is square root of 4, 3 is square root of 9,

4 is square root of 16, 5 is square root of 25.

Thus, C and D have the relation "*is a square root of*". From sets G and H, we observe that,

3 is one fifth of 15, 6 is one fifth of 30,

8 is one fifth of 40, 9 is one fifth of 45,

10 is one fifth of 50.

Thus, G and H have the relation "*is one fifth of*".

From the above discussion, we find that all the objects from the first set are matched with *unique* objects in the second set (*i.e.*, there cannot exist two or more objects in the second set for any object in the first set). There may be some objects in the second set which have no link with any object of the first set.

Note: **1.** A relation can be represented by matching diagram.

2. A mapping is a visual representation of a relation.

Domain, Co-Domain and Range of a Relation

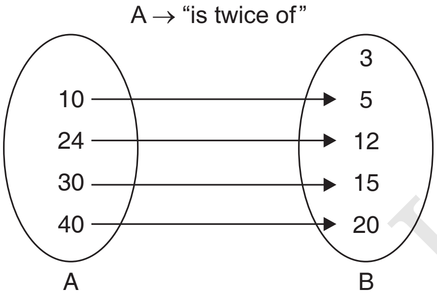
Domain

Domain of a relation is the set of all elements in the *first set* from the direction of the arrow diagram. It is represented by **D**.

Co-domain

Co-domain of a relation is the set of elements in the *second set* (*i.e.*, the whole second set) from the direction of the arrow diagram.

For example: Consider the following relation:



We observe that:

The set of elements in domain D = {10, 24, 30, 40}

The set of elements in co-domain is {3, 5, 12, 15, 20}

Range

In every relation, an element *a* in set A is related or (mapped onto) an element *b* in set B. Set A is domain and set B the co-domain to this type of relation, the element *b* in the co-domain is called the image of *a* in the domain A. The set of all the images of the domain is referred to as the range.

The range is therefore a subset of the co-domain and denoted by R.

For example, from the relation "*is twice of*" shown in the above diagram:

The set of elements in range R = {5, 12, 15, 20}

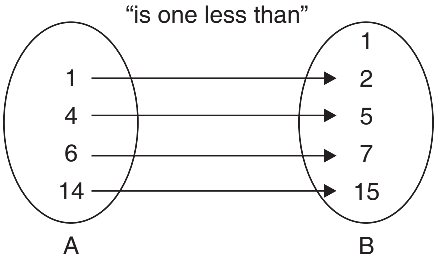
Example 2. *The following arrow diagram shows a relation between the two sets.*

Find the domain, co-domain and range.

Solution. The domain D = {1, 4, 6, 14}

The co-domain is {1, 2, 5, 7, 15}

The range R = {2, 5, 7, 15}.



Note:

1. Domain is the subset of elements in the first set from the direction of the matching diagram.
2. Co-domain is the set of elements in the second set (*i.e.*, the whole second set) from the direction of the matching diagram.
3. Range is a subset of the co-domain.

Relation as Ordered Pair

An *ordered pair* is a pair of objects taken in a specific order. An ordered pair is written by listing its two members in a specific order, separating them by a comma and enclosing the pair in parentheses. In the ordered pair (a, b), a is called the *first member* (or *component*) and b the *second member* (or *component*).

ACTIVITY 2

Suppose Daniel has two children—Felix and Ella. The ordered pairs in which the first component is father and the second component is a child are (Daniel, Felix) and (Daniel, Ella).

How will you interpret (Felix, Daniel) or (Ella, Daniel)?

Does the interpretation make sense?

Is (Daniel, Felix) = (Daniel, Ella)?

No, because the second member, *i.e.*, Felix and Ella are different.

Two ordered pairs are equal only when the corresponding components are equal.

Thus, $(a, b) = (c, d)$ only when $a = c$ and $b = d$.

Clearly, $(a, b) = (b, a)$ only when $a = b$.

Note: **1.** The two components of an ordered pair may be equal.

2. Remember that $\{a, b\} \neq (a, b)$, because $\{a, b\}$ is a set whereas (a, b) is an ordered pair.

Example 3. *If $(a, 2) = (3, 2)$, find the value of a.*

Solution. Since the ordered pairs are equal, the corresponding elements are equal.

Therefore, $a = 3$.

Example 4. *If $(x - 3, 5) = (2, y + 1)$, then find x and y.*

Solution. Here, $(x - 3, 5) = (2, y + 1)$

Equating corresponding components,

$$x - 3 = 2 \quad \text{and} \quad 5 = y + 1$$

$$\Rightarrow \quad x = 5 \quad \text{and} \quad y = 4$$

EXERCISE 7.1

1. Identify and write the relation between the following pairs of sets:

(*a*) Monrovia .... Liberia

(*b*) Mathematics .... High School

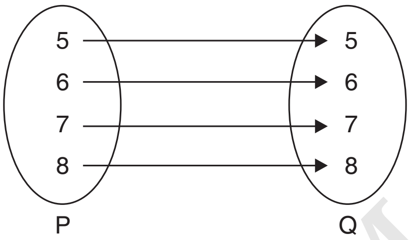
(*c*) English .... Liberia

(*d*) Nelson Mandela .... South Africa

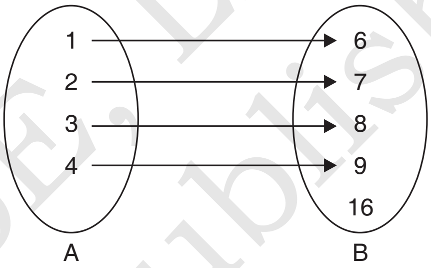
(*e*) L$ .... Liberia

2. Identify the following relations between the given pairs of sets:

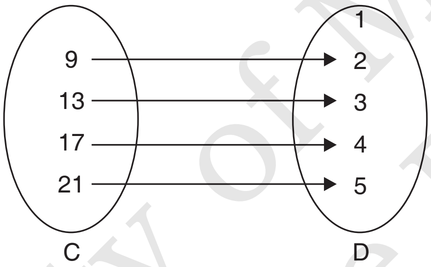
(*a*)



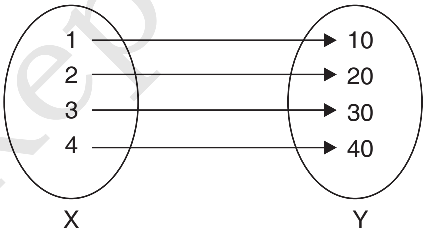
(*b*)



(*c*)



(*d*)



3. Given P = {Islah, Felix, Ella, Thomas},

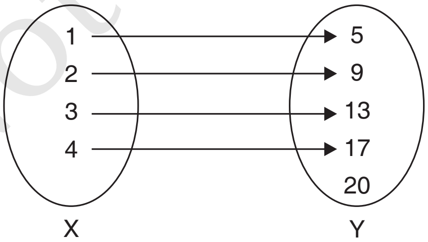
Q = {Days of the week}

The relation is "was born on".

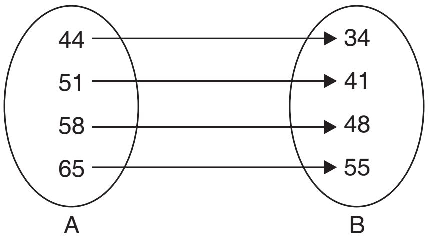
Draw an arrow diagram between the pairs of sets (assume a day of your choice for the birth of each).

4. Find the domain, co-domain and range of the following relations:

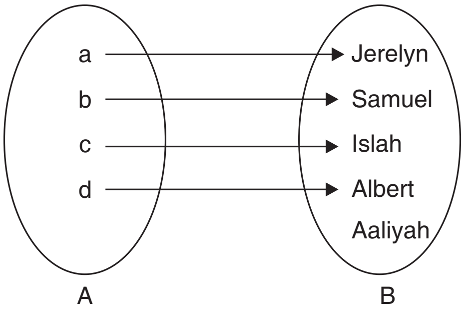
(*a*)



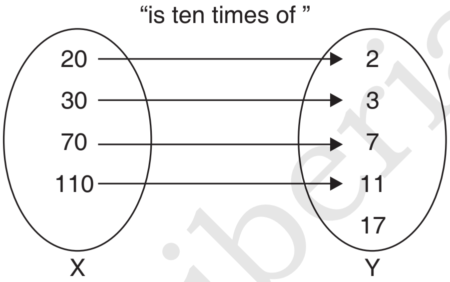
(*b*)



5. Write a set of ordered pairs of members that satisfy the following relation:



Q. 5



Q. 6

6. Write a set of ordered pairs of members that satisfy the above relation.

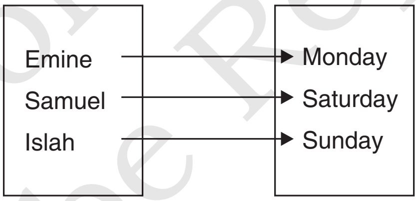
7. If $(x + 2, y - 4) = (5, 2)$, find the values of x and y.

7.2 TYPES OF RELATIONS

One-to-One Relation

In this type of relation, the first set (domain) is mapped to only one element on the second set (co-domain).

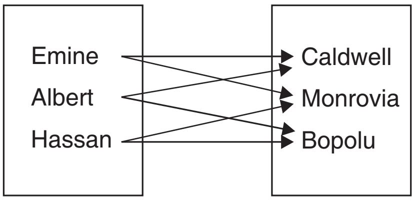
For example:



One-to-Many Relation

Here, each element in the domain is mapped to more than one element in the co-domain.

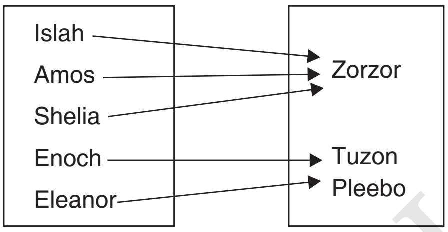
For example:



Many-to-One Relation

Here many elements in the domain are mapped to one element in the co-domain.

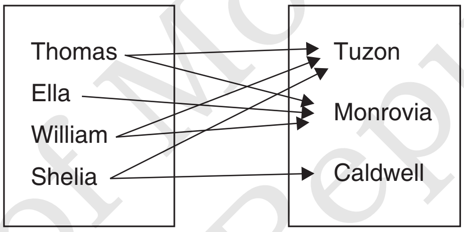
For example:



Many-to-Many Relation

In this relation, several elements in the domain are mapped to more than one element in the co-domain.

For example:



Example 5. *Let A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8}. Which of the following relations from A to B is one-to-one, one-to-many, many-to-one, many-to-many? Draw an arrow diagram in each case.*

(*i*) $R_1 = \{(3, 4), (4, 4), (5, 4), (2, 6)\}$

(*ii*) $R_2 = \{(2, 4), (3, 6), (4, 8)\}$

(*iii*) $R_3 = \{(2, 4), (2, 6), (2, 8), (5, 6)\}$

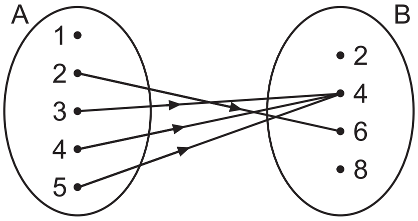
(*iv*) $R_4 = \{(2, 4), (3, 6), (3, 8), (4, 6), (4, 8), (5, 8)\}$

Solution.

(*i*) $R_1 = \{(3, 4), (4, 4), (5, 4), (2, 6)\}$

Here *many* elements 3, 4, 5 in the domain of R_1 are associated with one element 4 of B.

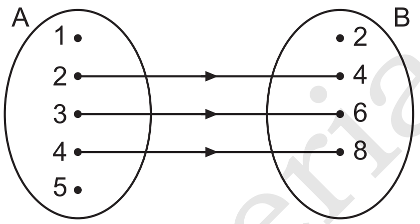
⇒ R_1 is a many-to-one relation.



(*ii*) $R_2 = \{(2, 4), (3, 6), (4, 8)\}$

Here every element in the domain of R_2 is associated with a unique element of B.

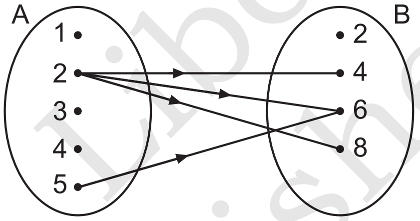
⇒ R_2 is a one-to-one relation.



(*iii*) $R_3 = \{(2, 4), (2, 6), (2, 8), (3, 6)\}$

Here an element 2 in the domain of R_3 is associated with many elements 4, 6, 8 of B.

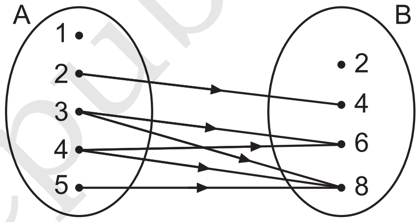
⇒ R_3 is a one-to-many relation.



(*iv*) $R_4 = \{(2, 4), (3, 6), (3, 8), (4, 6), (4, 8), (5, 8)\}$

Here many elements 3, 4 in the domain of R_4 are related to 6 and 8 respectively in B and 6 and 8 have many pre-images in the domain of R_4.

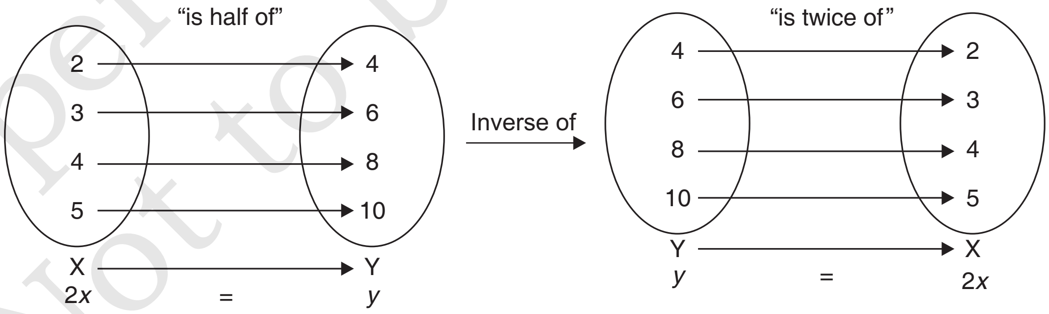
⇒ R_4 is a many-to-many relation.

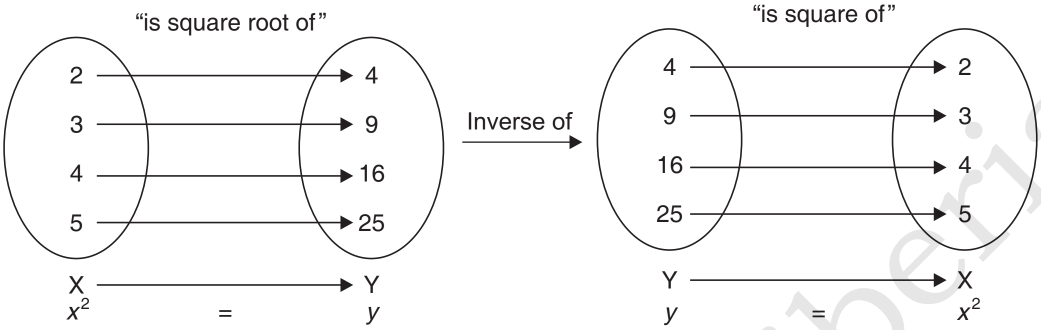


Rules for Mapping

We can state rules for mapping by using the inverse of the relation.

For example, consider the following relation:





Note: The inverse of a relation is possible only when the range equals the co-domain.

If $(x, y) \in$ R, we say that x is related to y.

To write the rule, we introduce a variable ordered pair (x, y) and for the rule, y is expressed in terms of x, (*i.e.*, the inverse relation).

For example, the rule of the mapping

$$\{(2, 4), (3, 6), (4, 8), (5, 10)\}$$

is the inverse mapping, which is "is twice" or y is two times x, (*i.e.*, $y = 2x$).

This may be illustrated in the following table:

Domain	2	3	4	5	x
	↓	↓	↓	↓	↓
Range	4	6	8	10	$y = 2x$

Example 6. *Find the rule for the mapping:*

$$\{(2, 1), (3, 4), (4, 7), (5, 10)\}.$$

Solution. The mapping is $\{(2, 1), (3, 4), (4, 7), (5, 10)\}$.

The rule of the mapping is the inverse mapping, which is "five less than thrice" or y is three times x minus five, (*i.e.*, $y = 3x - 5$). This may be illustrated in a table as shown below:

Domain	2	3	4	5	x
	↓	↓	↓	↓	↓
Range	1	4	7	10	$y = 3x - 5$

Finding the value(s) of *x* which make(s) relations undefined

Any expression of the form $\frac{f(x)}{g(x)}$ is *undefined* or *not defined*, if $g(x) = 0$.

Example 7. *Find the value of x which makes the mapping* $x \to \frac{9}{x+5}$ *undefined.*

Solution. Note that any fraction is undefined if the denominator is zero.

Therefore the mapping is undefined if

$$x + 5 = 0 \quad \Rightarrow \quad x = -5$$

EXERCISE 7.2

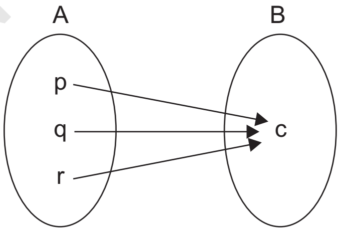
1. Determine the type of the relation in the following:
   (*a*) $R_1 = \{(1, 1), (2, 4), (3, 9), (4, 16)\}$
   (*b*) $R_2 = \{(1, 5), (2, 5), (3, 5), (4, 6)\}$
   (*c*) $R_3 = \{(3, 1), (3, 2), (3, 3), (1, 4), (2, 7)\}$
   (*d*) $R_4 = \{(1, 2), (3, 5), (3, 7), (4, 5), (4, 7), (5, 8)\}$
2. Find the rule for the following mappings:
   (*a*) $\{(2, 12), (3, 18), (4, 24), (5, 30)\}$ (*b*) $\{(1, 5), (2, 8), (3, 11), (4, 14)\}$
   (*c*) $\{(2, 18), (3, 25), (4, 32), (5, 39)\}$ (*d*) $\{(2, 16), (3, 81), (4, 256), (5, 625)\}$
   Show these in the form of tables also.
3. Find the value of *x* which makes the following relations not defined
   (*a*) $x \to \frac{1}{2x^2 - 5x + 2}$ (*b*) $x \to \frac{2x}{x^2 - 5x}$ (*c*) $x \to \frac{1}{4x^2 - 4x + 1}$

7.3 FUNCTIONS

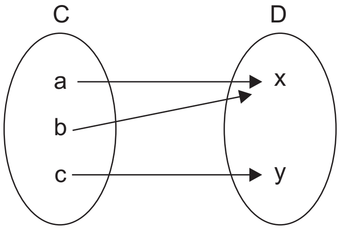
A *function* is a mapping in which each element in the domain is mapped onto one and only one member in the co-domain. Thus, for every element of the domain, there is exactly one image in the co-domain.

For example: (*a*) The following mappings are *functions*.

(*i*)

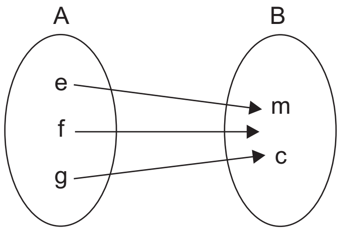


(*ii*)

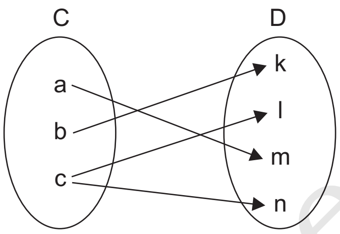


(*b*) The following mappings are *not functions*.

(*i*)



(*ii*)



In mapping (*i*), an element of the domain A (*i.e. f*) has no image.

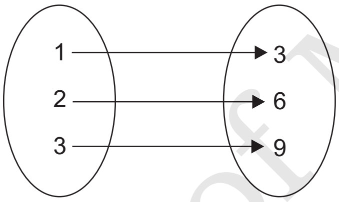
In mapping (*ii*), an element of the domain C (*i.e. c*) has more than one image (*i.e. l* and *n*).

One-to-one function

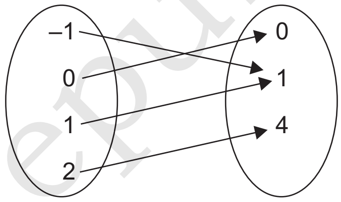
A one-to-one function is a function in which each element in the domain has only *one* image in the co-domain and each element in the co-domain is associated with only one element in the domain.

For example:

(*i*)



(*ii*)



In figure (*i*) above is a one-to-one function.

In figure (*ii*) above is *not* a one-to-one function since two elements of the domain {*i.e.* – 1 and 1} have the same image (*i.e.*, 1).

The Function Notation

If $f(x) = 2x + 3$, then by definition

$f(1) = 2(1) + 3 = 2 + 3 = 5$ (*i.e.* replace x by 1)

$f(2) = 2(2) + 3 = 4 + 3 = 7$ (*i.e.* replace x by 2)

$f(-3) = 2(-3) + 3 = -6 + 3 = -3$ (*i.e.* replace x by – 3)

Note that $f(1)$ means the image of 1 under the function f.

Example 8. *Let f be a function from A to B such that*

$A = \{1, 2, 3, 4\}$ *and* $B = \{3, 5, 7, 9, 11, 13\}$

If $f(x) = 2x + 1$, *find the range of* $f(x)$.

Solution. Here $x \in$ A and $f(x) \in$ B.

Given $f(x) = 2x + 1$

i.e., f image of x is $2 \times x + 1$.

Therefore, $f(1) = 2 \times 1 + 1 = 3$ $\quad f(2) = 2 \times 2 + 1 = 5$

$f(3) = 2 \times 3 + 1 = 7$ $\quad f(4) = 2 \times 4 + 1 = 9$

Range of f = set of f images of all elements of A.

$= f(A) = \{f(1), f(2), f(3), f(4)\} = \{3, 5, 7, 9\}$,

which is always a subset of co-domain B.

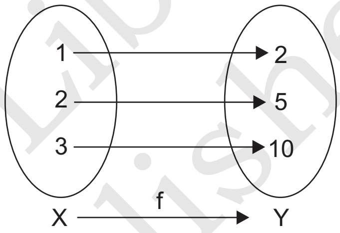
Example 9. *A function $f: x \rightarrow x^2 + 1$ is defined on the domain $\{1, 2, 3\}$. Use arrow diagram to show whether or not f is one-to-one.*

Solution. $f(x) = x^2 + 1$

$\Rightarrow$ $f(1) = 1^2 + 1 = 2$; $f(2) = 2^2 + 1 = 5$;

$f(3) = 3^2 + 1 = 10$

Since each member of the domain $\{1, 2, 3\}$ has distinct image in the co-domain, the function f is one-to-one.



EXERCISE 7.3

1. A function $g : x \rightarrow x^2 + 1$ is defined on the domain $\{-1, 0, 1, 2\}$. Use arrow diagram to show whether or not g is one-to-one.
2. A function is defined by $f : x \rightarrow \dfrac{2+x}{1+x}$.
   (*a*) Find the image of 1 and 2 under f.
   (*b*) What value of x makes the function undefined?
3. A function is defined by $f: x \rightarrow 2x - 3$ on the domain $\{-2, -1, 0, 1, 2\}$, find the range of the function.
4. The image of x of a function defined by $f : x \rightarrow 2x + 5$ is 15, find x.
5. A function $f : x \rightarrow ax + b$ is such that $f(1) = 9$ and $f(2) = 14$. Find;
   (*a*) the values of a and b (*b*) $f(-1)$
   (*c*) $f(0)$ (*d*) $f(-2)$

7.4 CHANGE OF SUBJECT

Subject of a formula is the variable which is expressed in terms of other variables involved in the formula.

When a formula involves more than one varables, we can express each variable in terms of the others, *i.e.*, we can make any variable the subject of the formula.

For example: Consider the formula $v = xyz$

Here v is expressed in terms of x, y, z so that v is the subject of this formula.

Writing the formula as $x = \frac{v}{yz}$, x is the subject

Writing the formula as $y = \frac{v}{xz}$, y is the subject

Writing the formula as $z = \frac{v}{xy}$, z is the subject

Thus, in a given formula, we can change the given subject to some other subject of our choice.

Example 10. *Make r the subject of the formula* $V = \frac{1}{3}\pi r^2 h$.

Solution. Given $V = \frac{1}{3}\pi r^2 h$

$$\Rightarrow \quad 3V = \pi r^2 h \quad \Rightarrow \quad \frac{3V}{\pi h} = r^2$$

$$\Rightarrow \quad r^2 = \frac{3V}{\pi h} \quad \Rightarrow \quad r = \sqrt{\frac{3V}{\pi h}}$$

which is the required formula with subject r.

EXERCISE 7.4

Change the subject of each of the following formulas to the letter given against them:

1. $I = \frac{PRT}{100}$, P

2. $C = 2\pi r$, r

3. $A = \pi r^2$, r

4. $V = 4\pi r^2$, r

5. $v^2 = u^2 + 2as$, s

6. $C = \frac{5}{9}(F - 32)$, F

7.5 GRAPHS OF LINEAR FUNCTIONS

Any function of the form $y = mx + c$, where m and c are constants is called a *linear function* or *linear relation.*

For example: $y = 3x + 1$, $5y = 7x - 2$, and $y = x$ are linear functions.

The graphs of any linear function is a *straight line*. To draw the graph of the linear function $y = 2x + 1$ in the interval $-2 \leq x \leq 4$, form a table of corresponding values of x and y. Put all the values of x from $x = -2$ to $x = 4$ (*i.e.* $x = -2, -1, 0, 1, 2, 3, 4$) into the given relation and find the corresponding y values i.e. when $x = -2$, $y = 2(-2) + 1 = -4 + 1 = -3$. When $x = -1$, $y = 2(-1) + 1 = -2 + 1 = -1$.

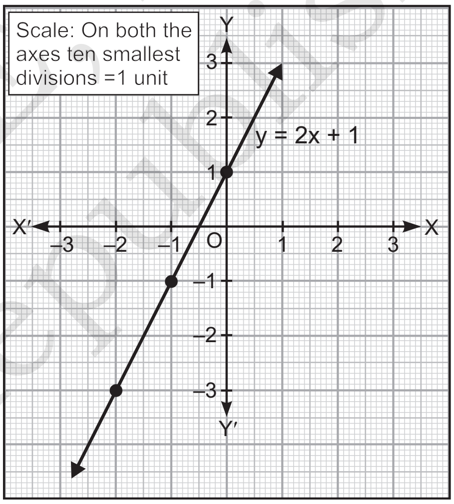
When $x = 0$, $y = 2(0) + 1 = 0 + 1 = 1$ etc. The table below shows the values of x and their corresponding values of y.

x	–2	–1	0	1	2	3	4
y	–3	–1	1	3	5	7	9

Note that you may use only three values of x in the interval given (*i.e.* $x = -2, 0, 4$) since the graph is a straight line.

Note: Here, we need only two points to draw a straight line. But more than two points enables you to check your calculations.

Now, plot the points (–2, –3), (–1, –1), (0, 1) etc. on the graph and draw a straight line through them as shown in the figure.



Example 11. *Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graphs for the straight lines* $x + y = 4$ *and* $3x - y = 8$ *on the same graph sheet. From your graphs find the coordinates of the point of intersection.*

Solution. Table for $x + y = 4$

$\Rightarrow \quad y = 4 - x$

x	– 1	1	2
y	5	3	2

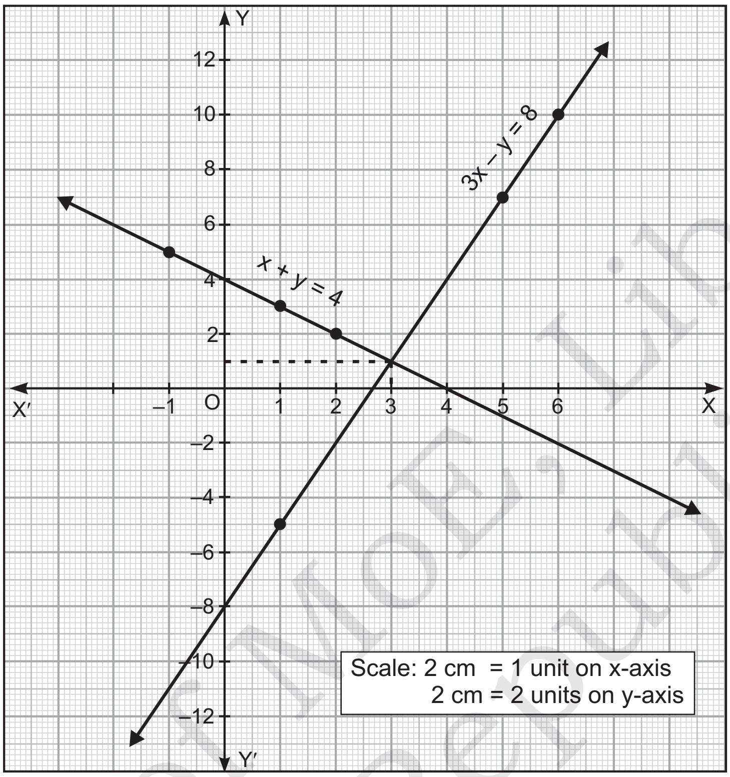
Table for $3x - y = 8$

$\Rightarrow \quad y = 3x - 8$

x	1	5	6
y	–5	7	10

Now, plot the above points on the graph and draw straight lines through them as shown in the figure.

From the graph, the point of intersection is (3, 1).



EXERCISE 7.5

1. Form a table of corresponding values of x and y given $y = 3x + 2$ in the interval $-2 \leq x \leq 4$. Plot the points on a graph sheet. Using a ruler draw a straight line through the plotted points.

2. Draw the graph of

(*a*) $y = 2x$ (*b*) $y = 2x - 2$ (*c*) $2y = x + 4$ (*d*) $x + 3y + 1 = 0$

(*e*) $x - y + 1 = 0$

3. Draw the graph of the following pair of linear functions.

(*a*) $y = x$ and $y = -x$ (*b*) $y = x - 1$ and $y = x + 4$

(*c*) $y = 2x + 3$ and $y = 2x + 5$ (*d*) $y = 3x + 11$ and $3y + x + 2 = 0$

7.6 GRAPHS OF QUADRATIC FUNCTIONS

Any function of the form $y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$, is called a *quadratic function* or *relation*. Thus a quadratic function is one containing x^2 as well as (perhaps) x and a number. $y = 3x^2 + 5x - 2$ is an example of a quadratic relation. The following illustrative examples will help us to draw quadratic graph.

The procedure is the same has been used for drawing linear graphs in the previous section.

Example 12. (*a*) *Copy and complete the table of values below for the relation*

$$y = x^2 - 2x - 3 \text{ for } -2 \leq x \leq 4$$

x	–2	–1	0	1	2	3	4
y	5		–3			0	

(*b*) *Using a scale of 2 cm to 1 unit on both the axes draw the graph of the relation.*

(*c*) *From your graph, find*

(*i*) *the positive value of y when x = –1.5*

(*ii*) *the positive value of x when y = –1.7*

Solution. (*a*) Given $y = x^2 - 2x - 3$, we can complete the table by substituting the given values of x in the relation to obtain their corresponding y values.

Note that the values of y for $x = -2$, 0 and 3 have already been given.

For $x = -1$, $y = (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$

For $x = 1$, $y = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$

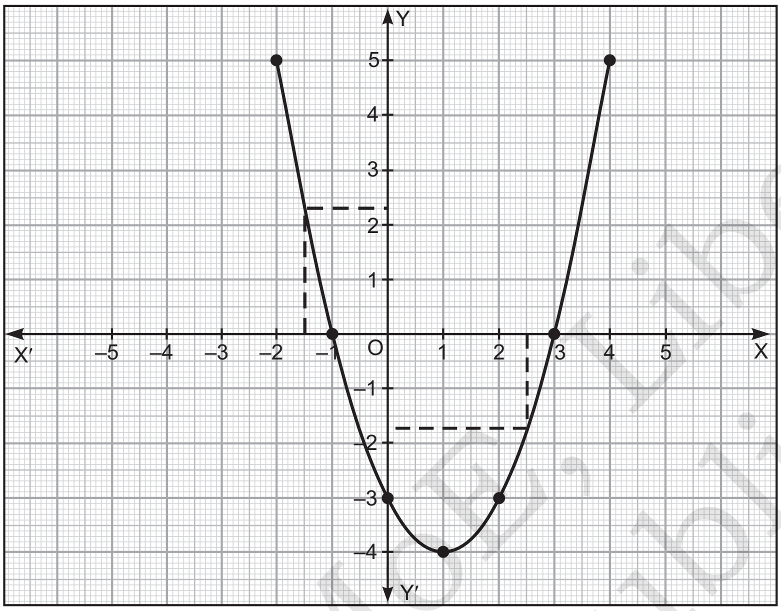
For $x = 2$, $y = (2)^2 - 2(2) - 3 = 4 - 4 - 3 = -3$

For $x = 4$, $y = (4)^2 - 2(4) - 3 = 16 - 8 - 3 = 5$

Hence the completed table is shown below:

x	–2	–1	0	1	2	3	4
y	5	0	–3	–4	–3	0	5

(*b*) The graph is shown in the figure.



(*c*) (*i*) From the graph, the positive value of y when $x = -1.5$ is **2.3.**

(*ii*) Also, the positive value of x when $y = -1.7$ is **2.5.**

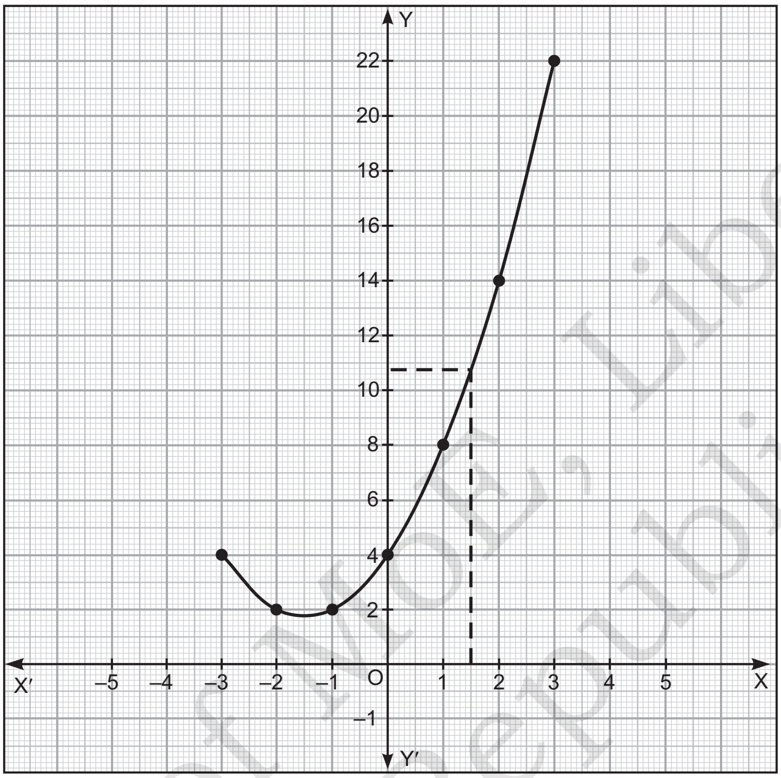
Example 13. (*a*) *Draw the graph of the relation* $y = x^2 + 3x + 4$ *for the interval* $-3 \leq x \leq 3$.

(*b*) *Use your graph to find the value of* y *when* $x = 1.5$.

Solution. Note that in this case an incomplete table is not given, therefore it is easier to set out your calculations as follows:

x	–3	–2	–1	0	1	2	3
x^2	9	4	1	0	1	4	9
$3x$	–9	–6	–3	0	3	6	9
4	4	4	4	4	4	4	4
y	4	2	2	4	8	14	22

The graph is shown in the figure.



(*b*) From the graph the value of y when $x = 1.5$ is **10.6.**

EXERCISE 7.6

1. Draw the graph of the function $y = x^2 + 2x$ from $x = -2$ to $x = 4$. Use your graph to find the values of y when x is 2.5.

2. The table below is a table of values for the function $y = x^2 - 6x + 5$ from $x = 0$ to $x = 6$.

x	0	1	2	3	4	5	6
y	5	0	–3	–4	–3	0	5

Draw the graph of the given function using the above table.

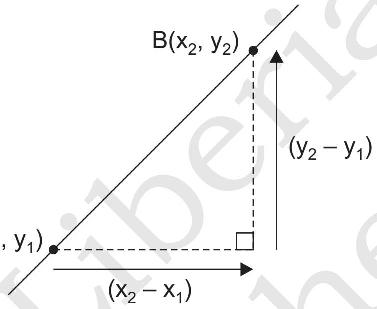
Use your graph to find the values of y when x is 1.5.

3. The table below shows values for the function $y = 12 - x - x^2$ from $x = -4$ to $x = 4$. Use your graph to find the value of x when y is 4.

x	–4	–3	–2	–1	0	1	2	3	4
y	–8	6	10	12	12	10	6	0	–8

7.7 GRADIENT OF A STRAIGHT LINE

Joining Two Given Points



Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two points on a straight line as shown in the figure.

The *gradient or slope* of the line joining points A and B is given by:

$$m = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

$$= \frac{\text{Difference in } y\text{-values}}{\text{Difference in } x\text{-values}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\Delta y}{\Delta x}$$

Note: The order of subtraction is very important.

Example 14. *Find the gradient of the line which passes through the points:*

(*a*) *A(1, 1) and B(7, 2)* (*b*) *P(2, 1) and Q(5, 5)*

(*c*) *L(3, – 2) and M(– 3, 4).*

Solution. (*a*) The gradient of line AB = $\frac{2-1}{7-1} = \frac{1-2}{1-7} = \frac{1}{6}$.

(*b*) The gradient of line PQ = $\frac{5-1}{5-2} = \frac{1-5}{2-5} = \frac{4}{3}$

(*c*) The gradient of line LM = $\frac{4-(-2)}{-3-3} = \frac{6}{-6} = -1$.

Finding Gradient of a Straight Line When its Equation is Given

To find the gradient, express the given equation in the form $y = mx + c$ and take the coefficient of x (*i.e.* m) as the gradient.

$$\underbrace{m}_{\text{Gradient}}x + \underbrace{c}_{\text{Intercept on } y\text{-axis}} = y$$

Example 15. *Find the gradient of the line with equation 2x + 4y = 7.*

Solution. Making y the subject of the equation,

$$2x + 4y = 7 \quad \Rightarrow \quad 4y = -2x + 7$$

$$\Rightarrow \qquad y = -\frac{1}{2}x + \frac{7}{4}$$

The coefficient of x is $-\frac{1}{2}$.

$\therefore$ The gradient of the line is $-\frac{1}{2}$.

EXERCISE 7.7

1. Find the gradient of the line which passes through the points:
(*a*) A(3, 1) and B(6, 10) (*b*) X(5, – 1) and Y(3, 5)
(*c*) P(3, – 2) and Q(6, 7)

2. Find the gradient of the lines passing through the following pairs of points;
(*a*) (0, 0) and (1, 3) (*b*) (1, 4) and (3, 7)
(*c*) (5, 4) and (2, 3)

3. Find the gradient of the following straight lines.
(*a*) $x + y = 4$ (*b*) $2x + y = 3$

4. Find the gradient of the line with equations:
(*a*) $3x + y - 2 = 0$ (*b*) $2x - 4y - 1 = 0$

7.8 DISTANCE BETWEEN TWO POINTS

Let A(x_1, y_1) and B(x_2, y_2) be any two points on a straight line as shown in the figure.

Using Pythagoras theorem,

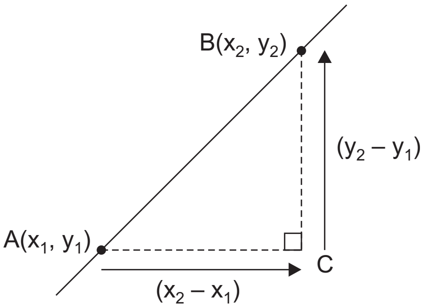
$$|AB^2| = |AC^2| + |CB^2|$$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore the length of the straight line joining the two points A and B is given by:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\Delta x)^2 + (\Delta y)^2}.$$



Example 16. *Find the length of the line joining the points P(4, 3) and Q(4, 7).*

Solution. $|PQ| = \sqrt{(4-4)^2 + (7-3)^2}$

$= \sqrt{(0)^2 + (4)^2} = \sqrt{16} = 4$ units

Example 17. *What is the distance between the points A(5, – 6) and B(2, 5)?*

Solution. $|AB| = \sqrt{[(5-(2)]^2 + [(-6-5)]^2}$

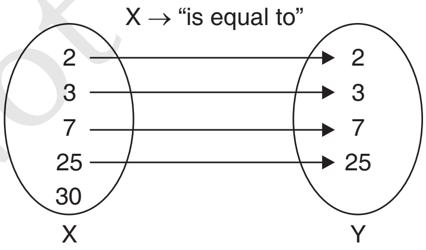
$= \sqrt{(3)^2 + (-11)^2} = \sqrt{9+121} = \sqrt{130}$ units.

EXERCISE 7.8

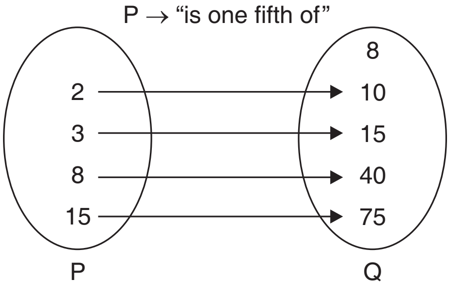
1. Find the length of the lines passing through the following pairs of points.
   (*a*) (1, 2) and (4, 6) (*b*) (3, 1) and (2, 0)
   (*c*) (4, 3) and (5, 2) (*d*) (7, 2) and (3, 6)
   (*e*) (0, 0) and (–1, –2).
2. Find the length of the line joining the following points:
   (*a*) P(1, 3) and Q(–2, 7) (*b*) A(–2, –2) and B(7, 10)
   (*c*) A(–4, 3) and B(5, 16) (*d*) C(–1, 2) and D(5, –6)

REVIEW EXERCISE

1. Identify and write the relation between the following pairs of sets:
   (*a*) 26th July .... Liberia (*b*) 4 16
   (*c*) 11 .... 20 (*d*) 100 50
   (*e*) 2^6 64
2. The following arrow diagram shows a relation between the two sets X and Y.
   Find the domain, co-domain and range.



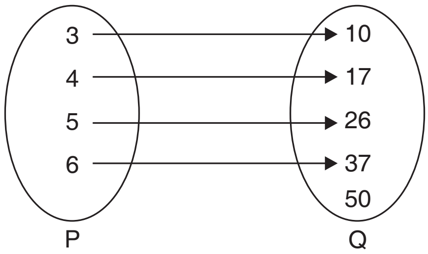
Q.2



Q.3

3. Find the domain, co-domain and range of the following relation.

4. Find the domain, co-domain and range of the following relation:



Q.4

5. Find the rule of the mapping: {(–1, 3), (–2, 6), (–3, 9), (–4, 12)}

6. Find the value of *x* which makes the following relations not defined

(*a*) $x \rightarrow \dfrac{x+1}{x^2-10x}$ (*b*) $x \rightarrow \dfrac{5x^2}{x^3-x^2}$

7. Let $f: x \rightarrow 2x-3,\ x \in \{-2, -1, 0, 1, 2, 3\}$. Find the range of *f*.

8. Given that $f(x) = px + q$, find the values of *p* and *q*, if $f(2) = 4$ and $f(4) = 10$.

9. The function *f* and *g* are defined as follows: $f: x \rightarrow \dfrac{x-1}{2}$ and $g: x \rightarrow 3x + 1$. (*a*) Evaluate $f\left(-\dfrac{1}{2}\right) + 1$ (*b*) Solve $f(x) = g(-2)$.

10. Change the subject of each of the following formulas to the letter given against them:

(*a*) $S = \dfrac{n}{2}(a + l), l$ (*b*) $S = 2\pi r(r + h),\ h$

(*c*) $S = ut + \dfrac{1}{2}at^2, a$ (*d*) $l = a + (n-1)d,\ n$

11. Draw the graph of the following pair of linear functions.

(*a*) $x = 2$ and $y = -1$ (*b*) $y = x + 2$ and $y = x$

12. Draw the graph of the function $y = 2x^2 + 3x - 7$ from $x = -2$ to $x = 4$.

13. Find the gradient of the lines passing through the following pairs of points;

(*a*) (2, 5) and (5, 9) (*b*) (–1, 2) and (2, –3) (*c*) (1, –3) and (5, 3)

14. Find the gradient of the line with equations:

(*a*) $5x - y + 3 = 0$ (*b*) $x + 7y - 5 = 0$

15. Find the length of the line joining the following points A(–5, 3) and B(5, 9).

16. Make *d* the subject of the formula $S = \dfrac{n}{2}[2a + (n-1)d]$.

MULTIPLE CHOICE QUESTIONS (MCQs)

1. If $x \rightarrow 3x - 4$, what is the image of -2?

(a) -10 (b) 2 (c) -2 (d) -1

2. What is the image of 3 in the mapping $x \rightarrow 3x + 7$

(a) 24 (b) 10 (c) 16 (d) 13

3. Find the image of -3 under the mapping $x \rightarrow 2(x + 3)$

(a) 0 (b) 12 (c) 2 (d) 6

4. What is the rule for the mappling below.

$$\begin{array}{cccccc} x & 0 & 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & 0 & 1 & 4 & 9 & 16 \end{array}$$

(a) $x \rightarrow \sqrt{x}$ (b) $x \rightarrow x + 2$ (c) $x \rightarrow x^2$ (d) $x \rightarrow x + 1$

5. Find the rule for the following mapping:

$$\begin{array}{cccccc} x & 0 & 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & -1 & 1 & 3 & 5 & 7 \end{array}$$

(a) $y = x^2 + 1$ (b) $y = 2x - 1$ (c) $y = 2x + 1$ (d) $y = x^2 - 1$

6. Find the rule for the mapping:

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & n \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 10 & 21 & 32 & 43 & 54 & \ldots \end{array}$$

(a) $n \rightarrow (10n + 1)$ (b) $n \rightarrow 10n$ (c) $n \rightarrow (11n - 1)$ (d) $n \rightarrow (7n + 3)$

7. Write the rule for the mapping

$$\begin{array}{ccccc} x & 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & 1 & 3 & 5 & 7 \end{array}$$

(a) $x \rightarrow 2(x - 1)$ (b) $x \rightarrow 2x + 1$ (c) $x \rightarrow 2(x + 1)$ (d) $x \rightarrow 2x - 1$

8. If $(x, 7) = (5, 7)$, then the value of x is:

(a) 5 (b) 7 (c) -5 (d) None of these

9. The gradient of the line passing through the points $(-5, -3)$ and $(4, 7)$ is

(a) $-\frac{10}{9}$ (b) $\frac{10}{9}$ (c) $\frac{9}{10}$ (d) $-\frac{9}{10}$

10. The gradient of the line of equation $7x - 5y + 8 = 0$ is

(a) $\frac{7}{5}$ (b) $-\frac{7}{5}$ (c) $\frac{5}{7}$ (d) $-\frac{5}{7}$

11. The domain and the range of the real function $f(x) = \dfrac{4-x}{x-4}$ is given by

(a) Domain = **R**, Range = {–1, 1} (b) Domain = **R** – {1}, Range = **R**

(c) Domain = **R** – {4}, Range = {–1} (d) Domain = **R** – {–4}, Range = {– 1, 1}

12. The length of the line joining the points (7, 4) and (–3, –1) is:

(a) $5\sqrt{2}$ units (b) $5\sqrt{3}$ units (c) $5\sqrt{5}$ units (d) $5\sqrt{7}$ units

RECAP AT A GLANCE

- A relation can be represented by matching diagram.
- Domain of a relation is the set of all elements in the *first set* from the direction of the arrow diagram.
- Co-domain of a relation is the set of elements in the *second set* from the direction of the arrow diagram.
- The range is a subset of the co-domain and denoted by R.
- An *ordered pair* is a pair of objects taken in a specific order.
- A *function* is a mapping in which each element in the domain is mapped onto one and only one member in the co-domain.
- *Subject of a formula* is the variable which is expressed in terms of other variables involved in the formula.

❑❑❑